6. a) $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} \dots (-1)^{n+1}\frac{x^n}{n}$

b) $\lim_{n\to\infty}\left|\frac{x^{n+1}}{n+1}\cdot\frac{n}{x^n}\right| = \lim_{n\to\infty}\left|x\cdot\frac{n}{n+1}\right| = |x|$

$x=1$ $\quad 1-\frac{1}{2}+\frac{1}{3}\dots$ conditional at $x=1$

converge by AST DUDES $\quad -1<x\leq 1$

abs conv for $-1<x<1$

$x=-1$ $\quad -1-\frac{(-1)^2}{2}+\frac{-1}{3}\dots \qquad -1-\frac{1}{2}-\frac{1}{3}\dots$

diverges harmonic $-\left(1+\frac{1}{2}+\frac{1}{3}\dots\right)$

c) $\ln\frac{3}{2} \approx \frac{1}{2} - \frac{(1/2)^2}{2} + \frac{(1/2)^3}{3} - \frac{(1/2)^4}{4} + \frac{(1/2)^5}{5}$ overestimate

$x=\frac{1}{2}$

$\epsilon < \frac{(1/2)^6}{6} = .002605$

d) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^{2n}}{2n} = \frac{x^2}{2} - \frac{x^4}{4} + \frac{x^6}{6} - \frac{x^8}{8}\dots$

Jan 30-7:30 AM

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots + (-1)^n\frac{x^{2n+1}}{(2n+1)!} + \dots$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \dots + (-1)^n\frac{x^{2n}}{(2n)!}\dots$

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^{n+1}\frac{x^n}{n}\dots$

Jan 30-8:02 AM

ch 9.

series $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 \dots$

sequence $\quad a_1, a_2, a_3 \dots$

geometric series $\sum_{n=1}^{\infty} a r^{n-1} = a + ar + ar^2 \dots$

$= \frac{a}{1-r}$ only if $|r|<1$

series conv. $|r|<1$

series div $|r|\geq 1$

Jan 30-8:12 AM

n^{th} term test: if seq does not converge to 0 the series diverges

{if seq → 0, need another test}

$\sum_{n=1}^{\infty}\frac{n}{n+1} = \frac{1}{2} + \frac{2}{3} + \frac{3}{4}\dots$

seq converges to 1

series diverges by n^{th} term test

$\sum_{n=1}^{\infty}\frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3}\dots$

diverges harmonic

seq conv to 0

series ?

$\sum_{n=1}^{\infty}\frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9}\dots$

converges by p series $p=2$

seq conv to 0

series ?

Jan 30-8:19 AM

p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ $p>1$, series conv
$p \le 1$ series div.

$\sum_{n=1}^{\infty} \frac{1}{n^3}$ conv. $p=3$

$\sum_{n=1}^{\infty} \frac{1}{3^n} = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} \cdots$ geo $r=\frac{1}{3}$

conv to $\frac{1/3}{1-1/3} = \frac{1/3}{2/3} = 1/2$ conv.

Jan 30-8:24 AM

ratio test use on $n!$ or a^n

$\sum_{n=1}^{\infty} a_n$ $L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$

$L<1$ series conv
$L>1$ series div
$L=1$??

comparison

direct. if $0 < a_n < b_n$
if $\sum b_n$ conv, $\sum a_n$ conv
if $\sum a_n$ div, $\sum b_n$ div

LCT if a_n & b_n grow at same rate ($\lim a_n/b_n = C \neq$ [illegible])
then $\sum a_n$ & $\sum b_n$ behave same

what to compare with?
keep dominant terms

Jan 30-8:29 AM

Integral test

$\sum_{n=1}^{\infty} a_n$ & $\int_1^{\infty} a(x)\,dx$ behave same

abs conv. if $\sum |a_n|$ conv
then $\sum a_n$ conv

alt series
AST if
1. signs alternate
2. $|a_{n+1}| < |a_n|$
3. $a_n \to 0$
then series conv.

remainder $< |a_{n+1}|$ AST

conditional conv. $\sum a_n$ conv but $\sum |a_n|$ div

Jan 30-8:36 AM

$\sum_{n=1}^{\infty} f(x,n)$ ~~$f(x)\, a_n$~~

ioc ratio test $L = |g(x)| < 1$

values of x where series conv.

$a < x < b$

check endpts
with different test

$\sum_{n=1}^{\infty} \frac{x^2}{2^n} = \frac{x^2}{2} + \frac{x^2}{4} + \frac{x^2}{8} + \cdots$

$\sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} \cdots$

Jan 30-8:41 AM

mac series

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 \dots$$

for x near 0

$$\frac{f^{(n)}(0)x^n}{n!}$$

Taylor series

x near a

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 \dots$$

$$\frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$\epsilon < \left| \frac{M(x-a)^{n+1}}{(n+1)!} \right|$$

Lagrange

M is upper bound for $\left| f^{(n+1)}(x) \right|$

Jan 30-8:46 AM

Jan 30-10:28 AM